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Various Methods of Tunnel Lining Design in Elastically Embedded Soil

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Abstract—In this research paper, the authors have dealt the various methods of Tunnel Lining design in elastically embedded soil and compared them. For the design of tunnel lining in elastically embedded soil , Zurabov and Bougayeva assume a continuous monolithic tunnel lining as a ring and thus take into account the support offered by the ground in the form of an elastic foundation. This method is considerably more accurate than the Davidov's method because has not considered the elastic subgrade reaction of subsoil. Davidov while calculating the stresses in ring neglects the effect of the lateral earth reaction and also the deflection at the horizontal diameter. Further A.M. Muir Wood and Morgan has also worked out empirical methods for solutions of ring design in elastically embedded soil. Out of the empirical methods in design of tunnel lining considered as a monolithic ring, Zurabov & Bougayeva method gives better approximate results, although for accurate results 3D Finite Element method may be adopted.

Index Terms— Bougayeva's Method of Tunnel Lining design, Davidov's Method of Tunnel Lining Design, Tunnel Lining design in Elastically Embedded Soil, Zurabov's Method of Tunnel Lining Design.

1.0 INTRODUCTION

The article compares the various methods of Tunnel Lining design in elastically embedded soil by empirical methods.

1.1 TUNNELS ELASTICALLY EMBEDDED IN THE SUBSOIL

It is an approximate method while considering the elastic embedding of the tunnel into the subsoil into full account. It is assumed that the embedded soil material around the tunnel is elastic and in it the reactions are proportional to penetrations, that is, the equation of Winkler-Schwedler is valid i.e.

$q = c.\delta_h$, where q = the pressure acting at right angles on the perimeter face of tunnel.

δ_h = the maximum horizontal deformation of the circular section of tunnel due to combinations of external loads and ground reaction and c = the coefficient of the subgrade reaction.

1.2 THE CIRCULAR TUNNEL IN ELASTIC GROUND: A.M. MUIR WOOD :

This design method is applied to changes of loading in the ground. Taking into account the stiffness of the lining and the loading transmitted to the ground around the extrados, starting from an applied normal load to the lining. Where P0 and P_v are horizontal and vertical earth reactions and Φ angle of repose respectively. Stress P is,

$P = p_v - \frac{P_o}{2}(1 - \cos 2\phi)$.

1.3 Morgan :

He showed that from consideration of change in curvature around the tunnel, the induced maximum bending moment can be calculated as follows:

$$M_{max} = +3\frac{u_o E I}{\eta^2 r_o^2}$$

But the corresponding maximum moment applied by the ground loading is,

$$M_{max} = +\frac{P_o \eta^2 r_o^2}{6}$$

Where , $E = E / \left(1 - \gamma_1^2\right)$ for a continuous lining.

The reduction of uo resulting from the stiffness of the lining leads to the following relationship between M_{max} & p_o

$$M_{max} .\left(\frac{1+ \lambda\eta^3 r_o^4}{9\,EI}\right) = +\frac{p_o \eta^2 r_o^2}{6}$$

Where $\frac{\sigma_{r\,(r=r_o)}}{u_o} = \frac{3E_c}{(1+\gamma)(5-6\gamma)r_o}$

Thus $M_{max} = +\frac{P_o r_o^2 \eta^2\ EI\,(1+\gamma)(5-6\gamma)}{6\,EI\,(1+\gamma)(5-6\gamma) + 2\,\eta^3 r_o^3 E_c}$

The stiffness ratio Rs represents the ratio of the stiffness of the tunnel lining to that of the surrounding ground. Thus can be calculated as,

$$Rs = \frac{3\,EI\,(1+\gamma)(5-6\,\gamma)}{E_c\,\eta^3\,r_o^3} = \frac{9\,EI}{\lambda\,\eta^3\,r_o^4}$$

And the reduction in bending moment to be carried by the lining is in relation to its flexibility and therefore we have.

$$M_{max} = +\frac{1}{6} p_o r_o^2 \eta^2\,[Rs/(1 + Rs)]$$

Where r_o = radius to extrados of tunnel lining.

p_o= excess of p on vertical axis over p on horizontal axis

p = normal pressure between ground and lining.

η= ratio of radius of lining centroid (= $\eta.r_o$) to that of extrados.

Rs= Stiffness factor. Ec= Young's modulus for ground.

E = Young's modulus for lining replaced by $\frac{E}{1-\gamma_1^2}$

γ = poissons ratio for lining. γ_1 = poissons ratio for ground

2.0 ZURABOV & BOUGAYEVA'S METHOD

At any point of the underground subsoil for the tunnel, the deformation is dependent on surrounding soil embedding pressure. The reaction provided by the soil is taken as a series of independently acting elastic supports. These are two basic assumptions of elastically embedded tunnel design. There are two similar empirical methods of designing – the first is that developed by ZURABOV & BOUGAYEVA and the second is that of DAVIDOV.

2. 1 ZURABOV & BOUGAYEVA'S METHOD: CALCULATIONS

For the design of the tunnel lining in elastically embedded subsoil, Bougayeva developed a simple empirical method which provides an approximate but quick solution.

This method takes the elastic embedment into account by determining the approximate values of the elastic reactions so that they satisfy the following criteria:

i)The condition of soil embedding pressure equilibrium and

ii)The condition which states that the displacements of the tunnel and of the subsoil embedment at the Springing line are equal. Thus only at these two diametrically opposite points, magnitude of the elastic reaction is correct. At any other point the magnitude and distribution of the reaction are arbitrarily assumed values, which, however, are close to their real values. For the distribution of the subgrade reaction the uniformly distributed vertical loading is considered. Typical values of this diagram are given by the following expressions:

If ρ is less than 45^0 ; the reaction is zero.(ρ is the angle between half ring top (Crown) and bottom and can vary from 0 degree to 180 degrees and Φ is the angle of repose.)

If 45^0 is less than 'ρ' is less than 90^0 ; $q.\delta = c\,\delta_v \cos 2\rho$

If 90^0 is less than 'ρ' is less than 180^0 then the value of q is $q.\delta = c\,\delta_v \sin 2\rho + c\,\delta_f.\cos 2\rho$

The structure is analyzed as a statically indeterminate structure to the third degree and, therefore, further two equations are required to determine the unknown values of c_v & c_f .These are the equations expressing the equilibrium of the forces, and the equation in which the horizontal deflection of the ring is equated to the compression of the soil at this point.

For analysis, the ring is reduced to a determinate structure by cutting it at the crown. Then by the application of the moment X_1 and forces X_2 and X_3 at the elastic centre, the reduced structure is made to act as the continuous, indeterminate structure. Because of symmetry the force X_3 will be zero. X_1 and X_2 are determined from the condition of zero displacement at the crown as per the following equations:

$X_1\, a_{11} + a_{01} = 0$

$X_2\, a_{22} + a_{02} = 0$

Because of the symmetry, the displacement factors are calculated for the half section only.

$$a_{11} = \int_o^{\pi} \frac{m_1^2\, ds}{EJ} = \frac{\pi r}{EJ}$$

$$a_{22} = \int_o^{\pi} \frac{m_2^2\, ds}{EJ} = 2\int_o^{\pi} \frac{.r^3 cos^2\phi\, .d\phi}{EJ} = \frac{\pi r^3}{2EJ}$$

The coefficients a_{01}, a_{02} are expressed as functions of $c\delta_v$ and $c\delta_f$.

The moment is determined first for the statically determinate half ring from the external loading. The half ring is divided into the sections and the moments for each segment are determined separately.

There are no reactions within the zone where 0 <ϕ< π Π/ 4.

$$Mo' = -\int_o^{\phi} pr_k \cos\rho (r\,.\sin\phi - r_k \sin\rho)\, d\rho = -\frac{1}{2}\, p\, r_k\, .r\, (2 - r_{k/r}).\, sin^o\, \phi$$

And adopting the notation. $\propto = \left(2 - \frac{r_k}{r}\right)$; $Mo' = -\frac{1}{2}\, p.r_k r._{\propto.\ sin^2\phi}$

The moment where π/4 <ϕ<π/2 can be worked as,

$$M"_o = -\frac{1}{2}pr_{k.}\propto .sin^2\phi - \int_{\pi/4}^{\phi} c\,\delta_r \sin(\phi - \delta\ )r_k\, .d\rho$$

$$= -\frac{1}{2}\, pr_k\, r\, .\propto .rsin^2\phi - c\,\delta_v r_k\, r \int_{\pi/4}^{\phi} \cos 2\rho \sin(\phi - \rho)\, .d\rho.$$

$$= -\frac{1}{2}\, pr_k\, r.\propto .sin^2\phi. - c\,\delta_v r_k\, r(\tfrac{1}{3}\cos 2\phi + 0.4714\,(\sin\phi - \cos\phi)$$

And in the third section where π/2<ϕ<π the moment can be computed as follows:

The forces transmitted from the upper sections are represented by the resultant of their vertical and horizontal components and a moment.

$$M_o''' = M + P_v r\,(1 - \sin\phi) + P_H r\cos\rho - \int_{\pi/2}^{\phi} c\,\delta_r \sin(\phi - \rho) r_k\, .d\rho$$

In this expression $M = -\frac{1}{2}\, p.r_k.r \propto - 0.1381\, c\,\delta_v.r_k\, .r$

Pv = $p.r_k. + \int_{\pi/4}^{\pi/2} c\ .\delta.cos\rho_{.r_k.d\rho} = r_k.(p + 0.1381\, c\, .\delta_v.)$ and the value of $P_H = \int_{\pi/4}^{\pi/2} c\ .\delta.sin\rho_{.r_k.d\rho} = 0.4714\, c\, .\delta_v.r_k$

Substituting these values into the expression for M_0 and integrating: $M"_o = -r_k\, r.p\,(\sin\phi + 0.5 \propto - 1) +$
$c\ \delta_v\,(-0.4714\cos - 0\,.19535\sin\phi + \frac{1}{6}\ \ cos2\phi + 0.5) +$
$c\ \delta_f\,(0.5 - .1667\cos 2\,\phi - 0.6667\sin\phi)$

Then writing the expressions for a_{01} and a_{02}

$$a_{01} = \int \frac{M_o m_1}{EJ}\, ds = \int \frac{M_o}{EJ}\, .ds$$

$$a_{02} = \int \frac{M_o m_2}{EJ}\, ds = \int \frac{M_o\, Y}{EJ}\, .ds$$

$$a_{01} = \frac{r}{EJ} \left(\int_o^{\pi/4} M'_o \, d\,\phi + \int_{\pi/4}^{\pi/2} M"_o \, d\phi + \int_{\pi/2}^{\pi} M"'_o \, d\,\phi \right)$$

And taking the appropriate substitution

$a_{01} = -\frac{r_k.r^2}{EJ}\,[p\,(1.1781 \propto -0.5708) + 1.0899\,c.\delta_v + 0.11875\,c\,\delta_f]$

$a_{02} = \frac{r^2}{EJ}\int M_o \cos\phi\, d\,\phi = -\frac{r^2}{EJ}\int_o^{\pi/4} M'_o \cos\phi\,.d\phi + \int_{\pi/4}^{\pi/2} M"_o \cos\phi.d\phi + \int_{\pi/2}^{\pi} M'''_o \cos\phi.d\phi$

Thus $a_{01} = -\frac{r_k.r^3}{EJ}\,[p\left(0.5 - \frac{\propto}{3}\right) - 0.82352\,c\,\delta_v - 0.11111\,c\,\delta_f]$

Writing these values of a_{01}, a_{02}, a_{11} and a_{22} we get :

$X_1 = r_k.r\,[p(0.375 \propto -0.18169) + 0.34694\,c\,\delta_v + 0.03778\,c\,\delta_f]$

$X_2 = r_k.r\,[p(0.21221 \propto -0.31831) + 0.52427\,c\,\delta_v + 0.07073\,c\,\delta_f]$

To determine δ_v and δ_f two additional equations must be established;

$\delta_v = \delta_{0v} + X_1\,\delta_{1v} + X_2\delta_{2v}$

And the sum of the vertical components of all the forces is equal to zero i.e. $\Sigma Y = 0$

The value δ_{0v} is the displacement of a point of the statically determinate structure located at the horizontal diameter and caused by the external loadings on the structure. Similarly δ_{1v} *and* δ_{2v} represent the displacement of the same point due to the action of the unit moment $X_1 = 1$ t-m and of the unit force $X_2 = 1$ t-m respectively.

$$\delta_{1v} = \frac{1}{E}\int M_v m_1 ds \quad where\ m_1 = 1\ tm;\ M_v = -r\cos\phi$$

Thus $\delta_{1v} = -\frac{1}{EJ}r^2.\int_{\frac{\pi}{2}}^{\pi}\cos\phi.d\phi = +\frac{r^2}{EJ}$ *and* $\delta_{2v} = \frac{1}{EJ}\int M_v m_2 ds$; $m_2 = -r\cos\phi$

$$\delta_{2v} = \frac{1}{EJ}\int_{\pi/2}^{\pi} r^3 cos^2\phi\, d\,\phi = \frac{\pi r^3}{4EJ}$$

$$\delta_{0v} = \frac{1}{EJ}\int M_o M_v\, ds = -\frac{r^2}{EJ}\int_{\pi/2}^{\pi} M'''_o \cos\phi\,.d\,\phi$$

$= \frac{r_k}{EJ}.r^3\,[0.5\,p\,(1-\propto) - 0.82807\,c\,\delta_v - 0.11111\,c\,\delta_f]$

On substituting these value we obtain from above;

$c.\delta_v\left(\frac{EJ}{c.r_k.r^3}. + 0.06937\right) = p\,(0.06831 + 0.04167 \propto) - 0.017778\,c\,\delta_f$

and

$(pr_k + 0.1381\,c\,\delta_v r_k + \int_{\pi/2}^{\pi} c\,r_k\,(\delta_v sin^2\rho + \delta_f cos^2.\rho)\cos\rho\, d\rho = 0$

or, $p - 0.1933\,c.\delta_v - \frac{2}{3}\,c.\delta_f = 0$

Writing the above equation into (c δ_v) equation and rearranging it we have.

$c.\delta_v = \frac{0.041671\,(1+\propto)}{(m+0.06416)}\,p$; $c.\delta_f = p\,[1.5 - 0.0122\frac{(1+\propto)}{m} + 0.06416]$

From where the expressions for X_1 and X_2 will become

$X_1 = p\,r_k.r\,[0.375 \propto -0.125 + 0.014\frac{1+\propto}{m} + 0.06416]$

$X_2 = p\,r_k.[0.21221(\propto -1) + 0.021\frac{1+\propto}{m} + 0.06416]$

Knowing X_1 and X_2 the stresses at any point of the ring are thus

$M = Mo + X_1 - X_2.\,r\cos\phi$

$N = No + X_2\cos\phi$

Writing into these equations the value of X_1 and X_2 ,the moments and normal forces can be derived at an arbitrary point of the ring which shall be as per the following expressions:

$M = p\,r_k.r\,[A \propto + B + C_1 n\,(1 + \propto)]$

$N = p\,r_k\,[D \propto + F + Gn\,(1 + \propto)]$

Thus the tabulated values for A, B, C, D, F and G.

$n = \frac{1}{m + 0.06416}$; $m = \frac{EJ}{r^3 c.r_k}$

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TABLE 1

BURAGOV'S CO-EFFICIENTS

	A	B	C	D	F	G
Φ=0	.1628	.0872	-.007	.2122	-.2122	.021
Π/4	-.025	.025	-.0084	.15	.35	.01485
Π/2	-.125	-.125	.00825	0	1	.00515
3Π/2	.025	-.025	.00022	-.15	.90	.0138
Π	.0872	.1628	-.0084	-.2122	.7122	.0224

2.2 DAVIDOV'S METHOD : CALCULATIONS

For the analysis of circular tunnel sections Davidov also developed and approximate method in which his assumption as to the distribution of ground reaction is similar to those made by Zurabov and Boygayeva . The ground reaction is expressed as a second degree trigonometric function. For the case of a uniformly distributed vertical load the external forces acting on a circular ring section i.e. the lateral active earth pressure e_1 is assumed to have a similar shape as the distribution of the lateral earth resistance e_2 .

i) The stresses in the ring are determined, neglecting the effect of the lateral earth reaction and also the deflection at the horizontal diameter.

Stress due to active earth pressure e_1 , $u_p = \int \frac{M_p M_H \, r \, d\phi}{EJ}$

Where M_p = the moment due to external load

M_H = the moment caused by the load

H = 1 t.

Thus $u_p = p \frac{r^4}{12EJ}$

ii) The stress, due to the horizontal load e_2 are calculated and the horizontal deflection u_2 of the structure is determined for this load condition as : $u_2 = -\frac{101}{1440} \cdot \frac{r^4}{EJ}$

iii) Next the compression of the soil, u_t , caused by the initial Horizontal pressure e is calculated at the line of the horizontal diameter.

$u_t = \frac{eH_i}{E_o}$, where e is the loading

H_i = thickness of the earth column considered to be compressed.

E_o = modulus of compressibility of soil.

The value of e is that value of the horizontal loading by which the compression of the soil just begins;

$u_p + u_2 \cdot e = 0$; $e = \frac{-u_p}{u_2}$

Substituting the value of u_p and u_2 we get ; e = 1.19 p

iv) The Then e_2 is determined, utilizing the expression that the deflection of the structure must be equal to the compression of the soil $u_p + e_2 u_2 = u_t \cdot e_2$

Thus $e_2 = \frac{u_p}{u_t - u_2} = \frac{\frac{p}{12} \cdot r^4}{\frac{EJ\,H_i}{E0} + \frac{101}{1440} \cdot r^4}$

Having determined e_2 all external loads are known and thus both the moments and the axial forces in the structure can be calculated.

Another main difference between Davidov and other methods considering the elastic embedment of the tunnel section is that he does not use the coefficient of subgrade reaction to determine the component of the soil but calculates in the way settlement analysis is done. When determining Hi , Davidov assumed an active zone limited by the condition that there is the maximum value of soil stresses due to lateral pressure which just attains 120% of the over burden pressure,

$\sigma_{max} = 1.2\, \sigma_{geol} = A.e$

The corresponding values of e and A are tabulated by him.

3.0 CONCLUSION

Thus Davidov eliminates the use of coefficient of subgrade reaction and it is not recommended even for approximate calculations. A.M. Muir Wood has extensively done research work on elastically embedded soil and tunnel lining design by his empirical method is more closer to accurate values than Bougayeva or Davodov's methods. Though for accurate results 3D model simulation and computer based software utilizing Finite Element Method should be adopted.

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